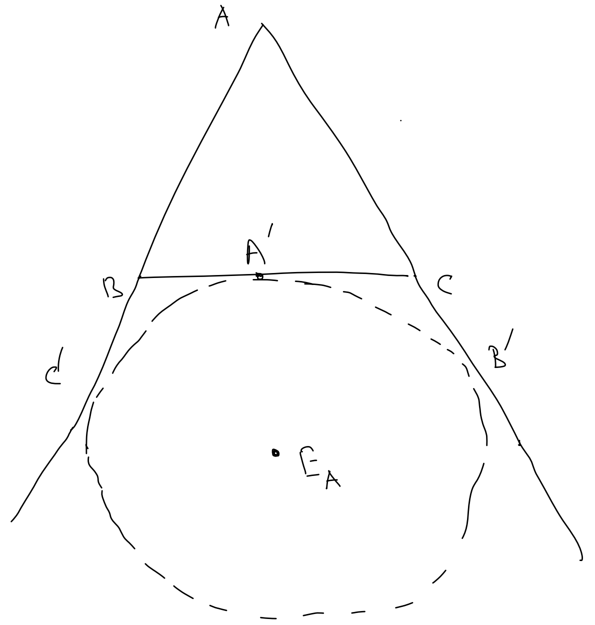
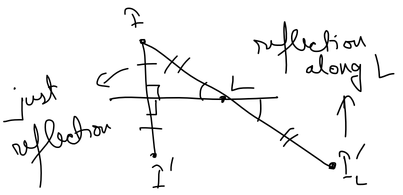
Geometry 6

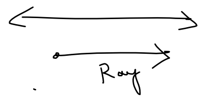
23 May 2024 18:28

E_A is the excentre of $\triangle ABC$ made opposite to A

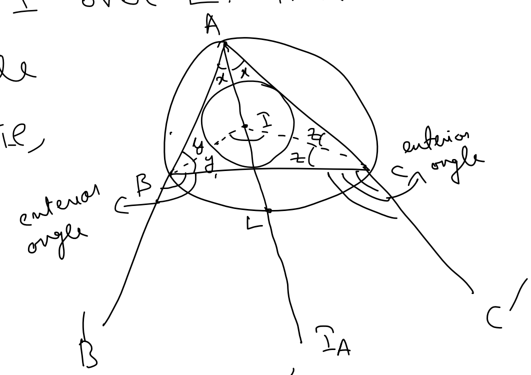




Lemma:- (Incentre/Excentre Lemma)



Let $\triangle ABC$ have incentre I. Ray AI meets (ABC) again at L. Let I_A be the reflection of I over L. Then



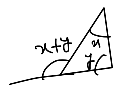
(a) The points I, B, C and I_A lie on a circle with diameter II_A and centre L, i.e.,

$$LI = LB = LC = LI_A$$

(b) Ray BI_A and CI_A bisect the exterior angles of $\triangle ABC$

to be proven later

This I_A is called the A-excentre



Proof:- (a) $x+y+z = 90^\circ$, $LI = LI_A$

$\angle CBL = \angle CAL = \angle IAC = x$ $\Rightarrow$ $\boxed{\angle LBI = \angle CBL + \angle CBI = x+y}$

$\angle LIB = 180^\circ - \angle BIA = \angle IBA + \angle IAB = \boxed{x+y = \angle LIB}$

$\Rightarrow LI = LB$

Similarly for LI, LC

LI = LB = LC = LI ... BI is the bisector

Similarly for LI, LC

$\Rightarrow LI = LB = LC = LI_A \Rightarrow II_A$ is the diameter as L is the centre

(b) $\angle IBI_A = 90^\circ$ $\qquad \angle IBC = y \Rightarrow \angle CBI = 90^\circ - y$

$\angle ICI_A = 90^\circ$ $\qquad \Rightarrow \angle I_A B B' = 90^\circ - y$

$\Rightarrow \angle CBB'$ is bisected by $I_A B$

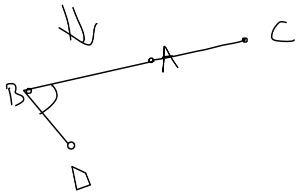
Similarly for $\angle BCI_A$

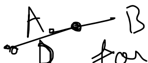
<u>Directed Angles</u> :- For any distinct points A, B, C, D in the plane, we have:-

(i) $\measuredangle APA = 0$

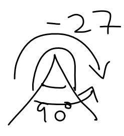
(ii) $\measuredangle ABC = -\measuredangle CBA$

(iii) $\measuredangle DBA = \measuredangle DBC$ iff A, B, C are collinear.





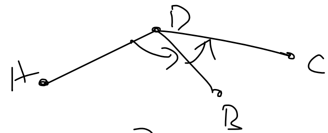
for $D=A$ case

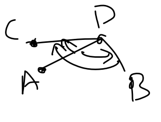


$-270^\circ = 90^\circ$

(iv) If $\overline{AD} \perp \overline{BD}$, then $\measuredangle ADB = \measuredangle BDA = 90^\circ$

(v) $\measuredangle ADB + \measuredangle BDC = \measuredangle ADC$

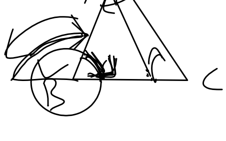




0°

In ΔABC,

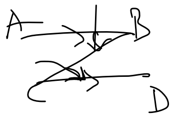
(vi) $\measuredangle ABC + \measuredangle BCA + \measuredangle CAB = 0^\circ$



(vii) In isosceles ΔABC,

$AB = AC$ iff $\measuredangle ACB = \measuredangle CBA$

(viii) If (ABC) has centre O, then, $\measuredangle AOB = 2\measuredangle ACB$



(ix) If $\overline{AB} \parallel \overline{CD}$, then $\measuredangle ABC + \measuredangle BCD = 0$